Two Modal Principles in Aristotle's *Metaphysics* Θ.4

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Target Passage

[It] is clear also that, ***if when A is the case it is necessary that B is the case, then also if A is possible it is necessary that B is possible****; for if it is not necessary that it is possible, nothing prevents it not being possible. Then let A be possible. Therefore, whenever A would be possible, if A were assumed, nothing impossible would have turned out; but then it is necessary that B is the case. But that was impossible. Then let it be impossible. Then if B is impossible, it is necessary that A is too. But then the first was impossible; so the second also. So if A were possible B will be also, if indeed they were so related that if A is the case it is necessary that B is the case. Then if, when A and B are related in this way, it were not the case that B is possible in this way, A and B will also not be related as laid down.* ***And if when A is possible it is necessary for B to be possible, then if A is the case it is necessary also for B to be the case.*** *For that B is of necessity possible, if A is possible, means this, that if A ever were the case both when and as it was possible then necessarily that too is at that time and in that way. (Prior Analytics Θ.4, 1047b14-30)*

Two modal principles

P1

If when A is the case it is necessary that B is the case, then also if A is possible it is necessary that B is possible.

P2

If when A is possible it is necessary for B to be possible, then if A is the case it is necessary also for B to be the case.

Current interpretations

P1a $(A \to B) \to (\Diamond A \to \Diamond B)$
P1b $\Box(A \to B) \to (\Diamond A \to \Diamond B)$
[P1b is equivalent to K]
P1c $\Box(A \to B) \to \Box(\Diamond A \to \Diamond B)$

P2a $(\Diamond A \to \Diamond B) \to (A \to B)$
P2b $\Box(\Diamond A \to \Diamond B) \to (A \to B)$
P2c $\Box(\Diamond A \to \Diamond B) \to \Box(A \to B)$

Three glaring issues with P2

1. Intuitively false due to obvious counterexamples
2. Leads to strange semantic and deductive consequences
3. Conflicts with Aristotle's aims

Consequences of accepting P2

P2a $(\Diamond A \to \Diamond B) \to (A \to B)$

- $p \to \Box p$ is a theorem, if P2a is an axiom. But then, R is vacuous, i.e. no world sees anything but itself, if it sees any world at all.
- $p \to \Diamond p$ is a theorem, if P2a is an axiom. But then, R is reflexive.
- But then no world sees anything except itself.

P2b $\Box(\Diamond A \to \Diamond B) \to (A \to B)$

- $\Diamond p \to \Box p$ is a theorem, if P2b is an axiom. But then, R is partially functional, i.e. any world sees at most one world.
- $\Diamond T$ is a theorem, if P2b is an axiom. Then, R is serial; any world sees at least one world.
- But then any world sees exactly one world.

P2c $\Box(\Diamond A \to \Diamond B) \to \Box(A \to B)$

- $\Box(p \to \Box p)$ is a theorem, if P2b is an axiom. But then, R is secondary vacuous, i.e. if wRv and vRu, then v=u.
- $\Box(p \to \Diamond p)$ is a theorem, if P2b is an axiom. But then, R is secondary reflexive, i.e. if wRv, then vRv.
- However, if we add $(\Box A \to A)$ as an axiom, R collapses into vacuity. It's hard to see how a metaphysical notion of modality can avoid axiom T.

K+P2c: The collapse argument

Furthermore, Kit Fine has shown that adding P2c to K results in a system that is deductively equivalent to the system $M_\Box$ defined by the axiom $\Box(A \leftrightarrow \Diamond A)$. If we add T, the new system is deductively equivalent to the system M defined by the axiom $(A \leftrightarrow \Diamond A)$.

- But M is the so-called 'Megarian thesis' – a rival view that identifies possibility with plain truth. In fact, this is a view that Aristotle is at pains to refute in the immediately preceding chapter of the *Metaphysics*.
- If Aristotle accepts T, his own view collapses into the view he takes himself to reject. Even if he doesn't accept T, he is still committed to taking the Megarian thesis to be necessary (even if somehow not true).

An alternative proposal

Tad Brennan: $P1 \leftrightarrow P2$ as a meaning-postulate.

1. P2 specifies the meaning (via truth conditions) of P1 in the metalangauge.
2. $(A \rightarrow B)$ iff $(\Diamond A \rightarrow \Diamond B)$, where $(\Diamond A \rightarrow \Diamond B) :=$ if A is true at α then B is true at α. The $\Diamond$ in the consequent serves as an anaphoric reference to the world picked out in the antecedent.
3. Then $\Box(A \rightarrow B)$ iff for any world α, if A is true at α then B is true at α.
4. Thus, P2 merely states the standard semantic clause for a strong conditional.
   - The non-standard reading of $\Diamond$ is independently interesting, but attributes an ambiguous use of 'possibly' to Aristotle and fails to make sense of his modal reasoning for P1.

Where from here?

- All in all, and alas and alack, Aristotle is plainly wrong in asserting P2.
- Existing paraphrases of P1 and P2 misrepresent the logical structure of these principles.

A fresh start

P1

If when **A** is the case **it is necessary that B** is the case, then also if **A** is possible **it is necessary that B** is possible.

P2

If when **A** is possible **it is necessary for B** to be possible, then if **A** is the case **it is necessary** also for **B** to be the case.

Necessity as Syllogistic Consequence

A syllogism is an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so. *By 'because these things are so' I mean that it results through these, and by 'resulting through these' I mean that no term is required from outside for the necessity to come about. (Prior Analytics 1.2, 24b18-22)*

If A is predicated of no B and B of every C, it is necessary that A will belong to no C. (26a1-2)

Upshot There's no (propositional) modal notion of necessity ($\Box$) required in either P1 or P2.

The structure of propositions

But first we must say that ***if it is necessary that B is the case if A is, then if A is possible, B will also be possible of necessity****. [...] Furthermore,* ***one should not understand 'B is the case if A is' as though, if some single thing A is so, B will be. For nothing is of necessity if just one thing is the case****; there must be at least two, as, for instance, when the premisses are related as we said for a syllogism.* ***For if C is said of D and D of F, C must also be said of F of necessity, and if each of the two is possible, so too is the conclusion. So if one assigned A to the premisses, B to the conclusion****, the result would be, not only that if A is necessary, B is necessary at the same time, but also that if A is possible, B is possible. (Prior Analytics 1.15, 34a6, 16-24)*

Upshots

1. A stands for a plurality of premises, while B stands for the conclusion of a syllogism.
2. Similarly, 'A is possible' and 'B is possible' stand for the premises and conclusion of a modal syllogism respectively.
3. The As and B (and their modal counterparts) must have the structure of predicative sentences.

Syllogistic analysis of possibility

1. **One-sided (M) possibility**
   - A is one-sided possible iff A is not impossible.
   - Formalised: $X_M Y$
   - $X_Q Y$ entails $X_M Y$
2. **Two-sided (Q) possibility**
   - A is two-sided possible iff A is neither impossible nor necessary.
   - Formalised: $X_Q Y$
   - $X_M Y$ *doesn't* entail $X_Q Y$
3. In his logical works, Aristotle is interested in Q-possibilities for the most part, and doesn't deal with any argument forms with M-premises. But the present, metaphysical, context concerns M-possibilities.

Metaphysical context: Analysis of capacities

- Cases: Humans have the capacity to walk; house-builders have the capacity to build; planets have the capacity to move.
- Capacity-to-possibility: It's possible *for* humans to walk; it's possible *for* house-builders to build; it's possible *for* planets to move.
- Possibility-to-predication: Walking *possibly holds of all* humans; house-building *possibly holds of all* house-builders; movement *possibly holds of all* planets.
  - Formalised: $X_{A_M} Y$
  - $\neg X_{A_M} Y$: $X_{O_N} Y$ [X necessarily does not hold of some Y; Y does not have the capacity X]

Two Modal Principles – the general version

P1

If $A \vdash B$, then $A \cup A_M \vdash B_M$

P2

If $A_M \vdash B_M$, then $A_M \cup A \vdash B$

- Both **P1** and **P2** are true in Aristotle's syllogistic. The proof of P1 is trivial, since B already entails B_M. Aristotle accepts the modal subordination of M-premises to X-premises. The proof of P2 proceeds by cases. (Note: Both P1 and P2 fail for two-sided possible premises.)

Restriction to capacity statements

- When the syllogistic schema is restricted to the figure in which both kinds of capacity-statements may be proved, i.e. statements of the form $X_{A_M}Y$ or $X_{O_N}Y$, we're only left with the first-figure.
- The first figure has the following form: X-Y, Y-Z $\vdash$ X-Z.
  Example:
  - Assertoric *Barbara* – $X_AY, Y_AZ \vdash X_AZ$
  - Modal *Barbara* – $X_{A_M}Y, Y_{A_M}Z \vdash X_{A_M}Z$

Two Modal Principles – the restricted version

In the first figure, the following are true:

P1*

If $A \vdash B$, then $A_M \vdash B_M$

P2*

If $A_M \vdash B_M$, then $A \vdash B$

- In the first figure, every valid modal schema for premises and a conclusion that is one-sided possible is also a valid assertoric schema (P2), and the converse (P1). (Note: Again, P2 fails for two-sided possible premises.)